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Damping of Sound Waves in Strong Centrifugal Field

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Abstract

A method for numerical calculation of the sound wave damping and dispersion law in a strong centrifugal field of the order of 10^6 g is considered. The damping is defined from the width of the resonance peak for different wave vectors. In the strong centrifugal field damping of the sound waves essentially exceeds the damping in the quiescent gas.

Keywords: fast rotation gas; sound waves; compressible gas;

Nomenclature

c	sound velocity	p	pressure
η	dynamic viscosity	T	temperature
ζ	second viscosity	M	molar mass of gas
k	thermal conductivity	R	gas constant
c_v	heat capacity at constant volume	ρ	density of gas
c_p	heat capacity at constant pressure	ω	frequency of the sound wave
r	radius of circular tube	ν	kinematic viscosity
χ	thermal diffusivity		

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1. Introduction

The most effective device for isotope separation is the gas centrifuge (GC). Typical GC rotors are 12-16 cm in diameter and rotate with linear velocity 600-700 m/s [1]. The centrifugal acceleration reaches $\sim 10^6$ g in these conditions. Strong centrifugal field creates elementary separation effect but it is very small. A secondary axial circulation flow is induced for increase of the GC efficiency. This secondary circulation is the main feature of the industrial GC.

There are two methods to drive the secondary circulation flow. These are heating of the gas (thermal circulation) and mechanical braking (mechanical circulation). Thermal circulation is induced by the temperature gradient on the rotor wall of the GC. The mechanical circulation is the result of the interaction of the fast rotating gas and a stationary scoop. A pair of scoops located at the end caps of the GC is used to remove enriched and depleted gas mixture from the GC. The sound velocity of the working gas UF^6 is about 86 m/s at room temperature. Therefore, the Mach number of the gas in the GC is close to 7 [4]. This results into formation of strong shock waves under interaction of the rotating gas and the stationary scoop which propagates along the rotational axis. The shock waves are damped rather quickly. They propagate as small amplitude waves in the largest part of the rotor. Therefore, it is reasonable to consider them in linear approximation.

It is well known that waves can produce the gas flow due to their absorption. This phenomenon is called acoustic flow and has been described by Rayleigh [2,3]. The flow arises as a result of momentum and energy transfer from waves to the gas due to the molecular viscosity. Therefore, we can expect that the waves produced by the scoops can induce an additional circulation. This is important for the physics of the GC. In this paper we propose and test the method for the numerical calculation of the damping length and dispersion law of the sound waves in the strong centrifugal field.

2. Damping of sound waves in quiescent gas

Let us consider molecular damping of sound waves in the quiescent gas. There are two well known expressions for the damping coefficients. First expression determines the volume damping coefficient [5]:

$$\gamma_1 = \frac{\omega^2}{2\rho c^3}\left(\left(\frac{4}{3}\eta+\zeta\right)+\kappa\left(\frac{1}{c_v}-\frac{1}{c_p}\right)\right). \quad (1)$$

The second one determines surface damping coefficient due to friction of the gas with a wall of the circular tube:

$$\gamma_2 = \frac{\sqrt{\omega}}{\sqrt{2}rc}\left(\sqrt{\nu}+\sqrt{\chi}\left(\frac{c_p}{c_v}-1\right)\right). \quad (2)$$

For the case of the quiescent gas we can obtain an equation of forced oscillation. The system of equations describing the dynamics of a viscous perfect gas is as follows

$$\frac{\partial\rho}{\partial t}+\nabla\left(\rho\vec{v}\right)=0, \quad (3)$$

$$\rho\frac{\partial v_i}{\partial t}+\rho v_k\frac{\partial v_i}{\partial x_k}=-\frac{\partial p}{\partial x_i}+\frac{\partial}{\partial x_k}\left(\eta\left(\frac{\partial v_i}{\partial x_k}+\frac{\partial v_k}{\partial x_i}-\frac{2}{3}\delta_{ik}\frac{\partial v_l}{\partial x_l}\right)+\varsigma\delta_{ik}\frac{\partial v_l}{\partial x_l}\right)+\xi_i\left(z,t\right), \quad (4)$$

$$\frac{\partial\rho h_{tot}}{\partial t}+\frac{\partial\rho h_{tot}v_i}{\partial x_i}=\frac{\partial p}{\partial t}+\frac{\partial}{\partial x_k}\left(v_i\eta\left(\frac{\partial v_i}{\partial x_k}+\frac{\partial v_k}{\partial x_i}-\frac{2}{3}\delta_{ik}\frac{\partial v_l}{\partial x_l}\right)\right), \quad (5)$$

where $h_{tot}=h(p,T)+v^2/2$.
The parameters of the gas can be presented as a sum of parameters of the quiescent gas marked by lower index "0" and a small perturbation marked by ` in the form

$$p = p_0 + p', \rho = \rho_0 + \rho', v_x = 0, v_y = 0, v_z = v_z', T = T_0 + T', \rho' = \frac{Mp'}{RT_0} - \frac{\rho_0 T'}{T_0}. \quad (6)$$

We take the force and gas parameters in the following form:

$$\xi = A(t)\sin(kz), p'(z,t) = P(t)\cos(kz), \rho'(z,t) = \Lambda(t)\cos(kz), v_z' = V_z(t)\sin(kz), T' = \tau(t)\cos(kz). \quad (7)$$

Then the system of equations (3-5) takes the form:

$$\frac{\partial \Lambda}{\partial t} + \rho_0 k V_z = 0, \quad (8)$$

$$\rho_0 \frac{\partial V_z}{\partial t} = kP - \frac{4}{3}\eta k^2 V_z + A(t), \quad (9)$$

$$\rho_0 c_p \frac{\partial \tau}{\partial t} = \frac{\partial P}{\partial t}. \quad (10)$$

The following second order differential equation can be obtained from the system of equations (8-10):

$$\rho_0 \frac{\partial^2 V_z}{\partial t^2} + \frac{4}{3}\eta k^2 \frac{\partial V_z}{\partial t} + \rho_0 k^2 c^2 V_z = \frac{\partial A}{\partial t}. \quad (11)$$

Its solution gives

$$V_z = \frac{A_1 \omega_f}{\rho_0 \sqrt{\left(\omega_0^2 - \omega_f^2\right)^2 + \frac{16}{9}\frac{\eta^2 k^4 \omega_f^2}{\rho_0^2}}} \cos\left(\omega_f t + \phi\right), \quad (12)$$

where

$$\phi = arctg\left(\frac{4}{3}\frac{\eta k^2 \omega_f}{\rho_0\left(\omega^2 - \omega_f^2\right)}\right). \quad (13)$$

The expression for the time-averaged energy of oscillation has the form:

$$\overline{E} = \overline{\frac{\rho_0 V_z^2}{2}} = \frac{A_1^2 \omega_a^2}{2\rho_0\left(\left(\omega_0^2 - \omega_f^2\right)^2 + \frac{16}{9}\frac{\eta^2 k^4 \omega_f^2}{\rho_0^2}\right)}. \quad (14)$$

We introduce the following notation:

$$\sigma^2 = \frac{16}{9}\frac{\eta^2 k^4 \omega_f^2}{\rho_0^2}. \quad (15)$$

Then the damping coefficient (1) can be represented as:

$$\gamma_1 = \frac{\sigma}{2c}. \quad (16)$$

σ characterizes the width of the resonance curve – dependence of the time-averaged energy of oscillation on the frequency of the driving force. From this curve one can determine the damping coefficient and the dispersion law of the waves.

Let us present pressure, axial velocity and temperature perturbations in equations (8)-(10) in the form

$$P = (P_1 + iP_2)e^{igt}, V_z = (V_1 + iV_2)e^{igt}, \tau = (T_1 + iT_2)e^{igt}. \quad (17)$$

Then we obtain the following system of ordinary differential equations:

$$\frac{MgP_{2,1}}{RT_0} - \frac{\rho_0 g T_{2,1}}{T_0} + \rho_0 k V_{1,2} = 0, \quad (18)$$

$$\rho_0 g V_{2,1} = kP_{1,2} - \frac{4}{3}k^2 V_{1,2} + \xi_2, \quad (19)$$

$$\rho_0 c_p T_{1,2} = P_{1,2}. \quad (20)$$

The resonance curve for the case of the quiescent gas is shown in Figure 1. The curve is calculated taking into account the volume damping only by the numerical method and analytically.

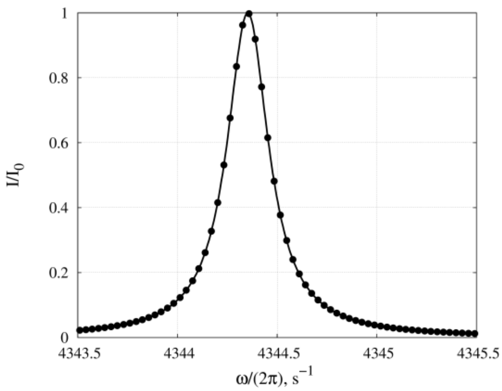

Fig. 1. Resonance curve for the volume damping of the waves. Solid line – theoretical dependence, dots – numerical results.

This curve allows us to calculate the damping length - the distance on which the velocity amplitude decreases *e* times. It is related to the damping coefficient as follows:

$$L = \frac{1}{\gamma}. \quad (21)$$

Dependence of the damping length on the wave vector has been obtained with the numerical method for the quiescent case. The results are shown in Figure 2. The dots show numerical data for the volume damping. The rhombs show numerical data for the volume and surface damping together, because in the numerical case we can't calculate only the surface contribution. Line 1 shows analytical dependence (1). It is clearly seen that the data coincide with high accuracy. Line 2 shows analytical dependence (2). Line 3 shows analytical dependence $\gamma=\gamma_1\gamma_2/(\gamma_1+\gamma_2)$ which takes into account the volume and surface damping.

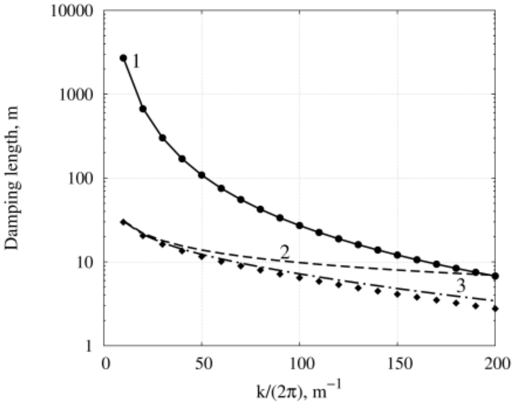

Fig. 2. Dependence of the damping length of the sound waves in quiescent gas versus wave vector. Dots – numerical data for volume damping, rhombs – numerical data for volume and surface damping, line 1 – analytical dependence for volume damping, line 2 – analytical dependence for surface damping, line 3 – analytical dependence for volume and surface damping.

One can see that the numerical results for the volume damping well agree with the analytical equation (1). There is discrepancy between the numerical and analytical calculations for the total (volume and surface) damping in the limits 5% at k<50. This difference achieves 15% at k=200. We assume that this happens because interference between the volume and surface damping was not taken into account. The results for the quiescent gas can be considered as a verification of the proposed numerical method.

3. Damping of the sound waves in strong centrifugal field

The system of ordinary differential equations describing the gas dynamics can be obtained in the case of strong centrifugal field from a system of equation describing the gas dynamics in cylindrical azimuthally symmetric coordinate system by following linearization as well [6,7]:

$$v_r = v_r', v_z = v_z', p = p_0 + p', \rho = \rho_0 + \rho', T = T_0 + T', p_0 = p_W \exp\left(\frac{M\omega^2}{2RT_0}\left(r^2 - a^2\right)\right). \quad (22)$$

We use the following notations

$$p(r,z,t) = (A_1(r) + iA_2(r))\sin(kz)e^{igt}, v_r'(r,z,t) = (U_1(r) + iU_2(r))\sin(kz)e^{igt}, \quad (23)$$

$$v_z'(r,z,t) = (W_1(r) + iW_2(r))\cos(kz)e^{igt}, v_\phi'(r,z,t) = (V_1(r) + iV_2(r))\sin(kz)e^{igt}, \quad (24)$$

$$T'(r,z,t) = (T_1(r) + iT_2(r))\sin(kz)e^{igt}. \quad (25)$$

Then we obtain the system of ordinary differential equations

$$\frac{MgA_{2,1}}{RT_0} - \frac{\rho_0 g T_{2,1}}{T_0} + \frac{(r\rho_0 U_{1,2})'}{r} - k\rho_0 W_{1,2} = 0, \quad (26)$$

$$\rho_0 g U_{2,1} - 2\rho_0 \omega V_{1,2} - \frac{M\omega^2 A_{1,2} r}{RT_0} + \frac{\rho_0 T_{1,2}\omega^2 r}{T_0} + A'_{1,2} = \eta\left(\frac{4}{3}\frac{\partial}{\partial r}\left(\frac{\partial(rU_{1,2})}{\partial r}\right) - k^2 U_{1,2} - \frac{kW'_{1,2}}{3}\right), \quad (27)$$

$$\rho_0 g V_{2,1} + 2\rho_0 \omega U_{1,2} = \eta\left(V''_{1,2} + \frac{V'_{1,2}}{r} - k^2 V_{1,2} - \frac{V_{1,2}}{r^2}\right) + \xi_1, \quad (28)$$

$$\rho_0 g W_{2,1} + kA_{1,2} = \eta\left(\frac{W'_{1,2}}{r} + W''_{1,2} - \frac{4k^2 W_{1,2}}{3} + \frac{kU_{1,2}}{3r} + \frac{kU'_{1,2}}{3}\right), \quad (29)$$

$$\rho_0 c_p g T_{2,1} - gA_{2,1} = \rho_0 \omega^2 r U_{1,2} + \lambda\left(\frac{T'_{1,2}}{r} + T''_{1,2} - k^2 T_{1,2}\right) + \theta_1. \quad (30)$$

We solve this system of equations numerically with the parameters listed in Table 1 which corresponds to the parameters of the Iguasu centrifuge [8] with different types of driven force and frequency.

Table 1. Parameters of the problem

Parameter	Value
M	0.352 g/mol
A	0.065 m
T_0	300 K
P_0	80 mm Hg
c_p	385 J/kg·K
H	$1.85{\cdot}10^5$ Pa s
Ω	$1700 \cdot 2\pi$ s^{-1}

We obtain the resonance curve which is shown in Figure 3. One can see several peaks. They correspond to the different dispersion laws. The existence of several dispersion laws in the strong centrifugal field was predicted in [9].

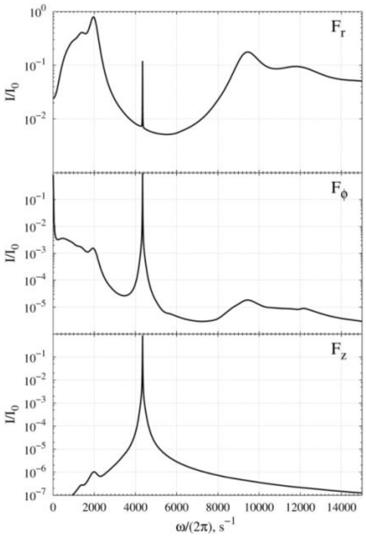

Fig. 3. Resonance curve for the volume damping of the sound waves for radial, azimuthal and axial driven force.

The damping length was determined for the waves with dispersion law $\omega=kc$. These waves give sharp peak in fig. 3. Figure 4 shows dependence of the damping length on the wave vector of sound wave in the case of strong centrifugal field. Dots correspond to the volume damping and rhombs correspond to surface and volume damping together. It can be seen that the damping length in the case of strong centrifugal field much smaller than in the quiescent gas. This is due to the larger radial pressure gradient which increases the radial gradient of the velocity.

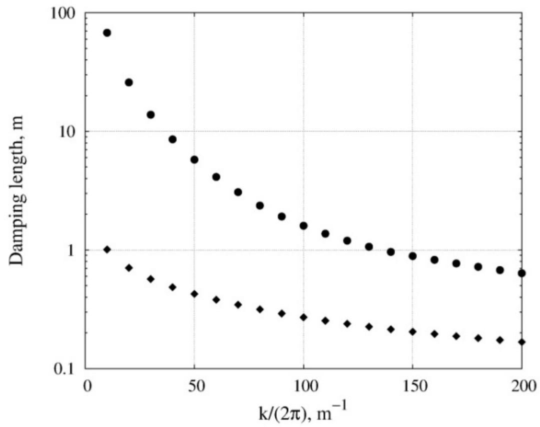

Fig. 4. Dependence of damping length of sound waves in strong centrifugal field versus wave vector of driving force. Dots – numerical data for volume damping, rhombs – numerical data for volume and surface damping.

4. Conclusion

In this paper the method for the numerical determination of the damping length of the sound waves in the gas is proposed. It was verified for the case of the quiescent gas.

The resonance curves for the waves in strong centrifugal field were obtained for different wave vectors. Several types of the waves were found as predicted theoretically. The dependence of the damping length on the wave vector was obtained for the waves with dispersion law $\omega=kc$. The damping length in this case is an order of magnitude less than in quiescent gas.

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